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COSMOLOGY Q & A

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Cosmology Q & A

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Cosmology is the study of the universe as a whole. While not short on ambition, it does engender confusion. After reviewing the basics, I'll answer some of cosmology's "dinner party" questions.

A Simple Universe

The real scandal of cosmology is its simplicity. Let me explain. Einstein's General theory of Relativity (GR) relates the geometry of spacetime to the energy (of all forms, including matter) it contains. Roughly, you tell me where the stuff is and I'll tell you how space and time intertwine and contort. This *spacetime curvature* manifests itself as gravity. Gravity is not a force that curves trajectories. Rather, objects moving under gravity travel along locally straight lines (*geodesics*); energy distorts the very structure of spacetime beneath them. Gravity doesn't turn the steering wheel; it banks the curve.

Peeking at GR's cogs and springs, which turn energy into spacetime geometry, we find a system of 10 coupled, non-linear, partial differential equations. For the mathematically uninclined, this is a bit like hearing the dentist say "root canal", or the mechanic say "head gasket" — I'm not sure what those words mean, but I know that pain is coming.

Applying these equations to the whole universe, then, seems a sure road to insanity, a task for a masochist. So cosmologists in the early days of relativity did what any good physicist does — they oversimplified. They assumed a very strong symmetry: on sufficiently large "cosmological" scales, the universe is the same everywhere (*homogeneity*) and looks the same in all directions (*isotropy*). This assumption was given the lofty title of "the cosmological principle" but let's be frank — it's an optimistic guess, a toy model, a practice problem. Like the frictionless pulley or the infinite plane conductor, the real universe surely can't be *that* simple. (In 1953, Herbert Dingle memorably warned his cosmological colleagues not to aggrandize a mere assumption: "call a spade a spade, and not a perfect agricultural principle".)

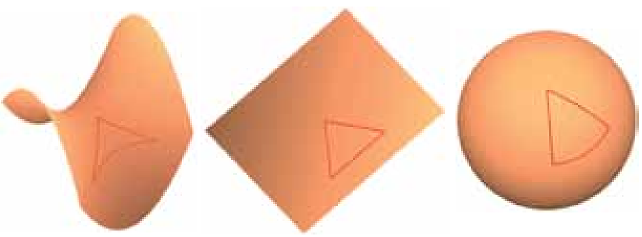

Figure 1: Three curved spaces, each with a triangle built from locally-straight lines (geodesics). In the negatively-curved space (left), the triangle's internal angles add up to less than 180 degrees, while in the positively-curved space (right) more than 180 degrees are to be found.

This ludicrously simple model of the universe, known as the Friedmann-Lemaitre- Robertson-Walker (FLRW) model, turns out to be all we have ever needed. After nearly 100 years of modern cosmology, during which various complications to the FLRW model have been investigated, none have improved on the original. The universe is just about as simple as we could have hoped.

Curving and Expanding: The FLRW Model

The cosmological principle makes my job as a cosmologist immeasurably easier. Einstein's formidable field equations reduce to a special case called the Friedmann equations, comprising two first order ordinary differential equations. Continuing our analogy, it's like hearing the dentist say "toothbrush"—*that* I can handle!

The FLRW model describes two things about the spacetime of the universe as a whole. The first is the *geometry* of space. In the early 1800's, Nikolai Lobachevsky, the "Copernicus of Mathematics", showed that there is nothing unique about Euclidean or *flat* geometry, that is, the familiar geometry of high school where triangles have angles that add up to 180 degrees and parallel lines never meet. Mathematics tells of two other possible homogeneous 3D geometries, illustrated in Figure 1. The universe as a whole could be positively curved, like a 3D version of the surface of a sphere. Or else it could be negatively curved, somewhat resembling a saddle. (Unfortunately, the 2D version of this geometry is mathematically impossible to represent in three dimensions. If you can imagine 6-dimensional shapes, let me know what it looks like.)

This curvature of space is not merely abstract mathematics. It is measurable. If you find yourself in a spatially-curved universe, and have handy a really big triangle and a lot of spare time, you will measure that its internal angles don't sum to 180 degrees. It matters not what the triangle is made of; the curvature is written into space itself.

GR tells us how the geometry of the universe depends on its energy density. Overfill your universe and it will be positively curved—think of the aftermath of a Christmas dinner. If underfilled, negatively curved. On the dividing line, flat Euclidean geometry holds, as seems to be the case in our universe (on cosmological scales).

Second, the model describes the *scale* of space. Think of a model train — double the scale and all the parts double in size. In the case of the universe, it is not the contents of the universe but the scale of *space itself* that changes. On cosmological scales, beyond the reach of binding forces like gravity and electromagnetism, the distance between any two galaxies increases in proportion with a(t), the relative scale of the universe at time t.

Again, GR tells us how energy dictates the evolution of the scale of the universe. I'll risk a modicum of mathematics at this point. Here is the equation to solve,

$$\left(\frac{da}{dt}\right)^2 + K_0 = \rho a^2 \tag{1}$$

This mirrors the "CURVATURE = ENERGY" form of Einstein's GR field equation. The first term on the left is the rate at which the scale of the universe changes with respect to time (squared). The second is a constant proportional to the curvature of space. The right hand side comes from the energy side total energy density of the universe ρ. Note that all these quantities are averaged over a large region of space. [K_0 is not the infamous cosmological constant Λ, which can be treated as a form of energy with constant energy density and bundled into ρ. For the specialists, I've chosen my units to set a few physical constants to unity: $8\pi G/3 = 1$, and $c = 1$.]

As it stands, we cannot solve Equation 1 because we haven't specified how the total energy density of the universe ρ changes with the scale a. For ordinary matter, most of its energy is rest-mass energy ($E = mc^2$), which isn't affected by the expansion. If the universe doubles in scale, the same amount of energy is spread over a volume that is eight times larger. Thus, for the matter component, ρ dilutes as a^{-3}, and the corresponding term in Equation 1 decreases as a increases. Ordinary matter decelerates the expansion of the universe. For a form of energy whose kinetic energy is much greater than its rest-mass energy (e.g. photons), there is an additional effect due to the stretching of the de Broglie wavelength of each particle. Thus, for the radiation component, ρ dilutes as a^{-4}, which makes the universe decelerate even faster.

With that background, let's answer a few questions.

Is space expanding, or are galaxies just moving away from us?

GR and Newtonian gravity make the same predictions in the "weak field" regime, which for the universe corresponds to cosmologically "nearby" distances. GR describes an expanding space, while the Newtonian model portrays galaxies receding through absolute, static space. Locally, these look the same.

Since we believe from experiment and observation that GR is the more correct theory and Newtonian gravity the approximation, the expanding space picture is the more correct picture. Further, GR can handle global situations that Newton's theory can't. For example, the universe could be finite but unbounded - like the surface of the earth, which has a finite area but no edge. In that case, the total volume of the universe, the total amount of space, really does increase with time. There is, quite literally, more space. You could fit more oranges into the universe today than you could yesterday. An infinite universe can't get bigger, of course, but it seems natural to describe its expansion in the same way — you could fit more oranges between any two galaxies expanding with space.

Is everything getting bigger?

No, since then we couldn't tell. There are two reasons why some objects fail to expand in step with the universe as a whole. On small scales, the universe is not perfectly homogeneous. Clumps of matter experience the mutual attraction of gravity; if sufficiently dense, the matter and its spacetime ceases to expand. On very small scales, other forces like electromagnetism will hold objects to be a constant physical size.

You said above that ordinary matter and radiation cause the expansion of the universe to decelerate. But our universe is accelerating! How? What is the universe made of?

Answer that and collect your Nobel prize. The conundrum is rather straightforward. Suppose you knew of only heavier-than-air gases, and saw a helium balloon floating into the sky. No known gas will do that! What's in that balloon?

The universe is doing something that the familiar forms of matter-energy — protons, neutrons, electrons, photons, neutrinos — cannot do. From the Equation 1, we need a form of matter whose energy density dilutes slower than a^{-2}. It could be the cosmological constant, a term that naturally appears in the field equation of GR, but is unconstrained by the theory itself. It could be energy associated with space itself, which (like the cosmological constant) would have a constant energy density. In that case, the rate at which the universe expanded would increase with the scale of the universe, leading to runaway exponential expansion. A form of energy that can make the expansion of the universe accelerate is known generically as "dark energy". We know that it makes up about 70% of the total energy-density of the universe. We know that it behaves approximately like a cosmological constant. But that's about all we know.

Note that all this is in the context of the FLRW cosmological model. We postulate a new type of energy, not a new model. It is possible that we should instead revise Einstein's GR equations. Such models are being investigated but seem *ad hoc*. Ultimately, data will decide.

Dark Energy? Is that like Dark Matter?

Dark matter and dark energy are called "dark" because of their effect on telescopes. Stars, for example, are well-loved by astronomers — compact, long-lived, gloriously luminescent and concentrated into galaxies. Dark matter and dark energy give us nothing. We deduce their presence not from the light they emit, but from their gravitational effect on luminous bodies. As we saw above, dark energy's repulsive gravitational effect explains the acceleration of the expansion of the universe. We need dark matter's attraction to hold galaxies together — if they only contained the matter we can see, galaxies would be rotating too fast to be stable. They would fly apart.

We know from cosmological data that dark matter makes up 25% of the energy density of the universe. But we don't know what it is — another Nobel prize on offer. For those keeping score, that's 95% of the universe whose identity remains a mystery.

How big is the universe?

There are a few questions here. Note that until now I have referring to the scale of the universe, rather than the size, to emphasise that it is a relative measure. If we're thinking about the size of the observable universe, the question is how far light has travelled since the beginning of the universe. This is not simply ct_0, where t_0 is the age of the universe, because the universe is expanding.

In Figure 2, I take a metre-long stride at time t_1 from A to B, and another such stride later at t_2 from B to C. At t_3,

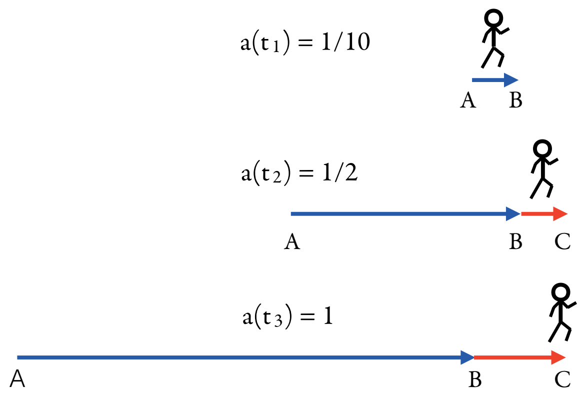

Figure 2: Walking across an expanding universe. I take a metre-long stride at times t_1 and t_2. From t_1 to t_2 to t_3 the universe expands in relative scale (*a*) from 1/10 to 1/2 to 1. The earlier step at t_1 has expanded to 10 metres by t_3, and so contributes more to the total distance (AC) than the later step.

the universe has expanded by a factor of 10 since t_1 and a factor of 2 since t_2. AB is now 10 metres, while BC is just 2 metres. Strides taken while the universe is small count more towards the total distance today between start and finish. For light, which moves at a constant speed, each time interval dt adds a distance $ds = cdt/a(t)$ to the final distance, where $a(t)$ is the scale of space at time t. Integrate ds from the beginning of the universe until today and you've got yourself the observable size of the universe. You'll need to solve Equation 1 to get $a(t)$. The observable limit of our universe is 46 billion light years away, larger than ct_0 by a factor of about three.

How big is the universe *really?*

We don't know. GR tells us how spacetime is curved (geometry), but not how it is connected (topology). If space is positively curved, then it is finite. If space is flat or negatively curved, then it *could* be infinitely large. Or it could be finite. For example, the "flat torus", as its name suggests, has a flat geometry, finite size, and the topology of a donut. If our assumption that the cosmological principle holds everywhere in the universe is wrong, then all bets are off. In any case, we can't see more than the observable universe, so we're theorizing.

If the universe were finite, could I see the back of my own head?

In principle, yes. However, it will take time for the photon to get all the way around the universe. If the universe has a finite lifetime, the photon might not get back to us before the end of the universe.

Is space expanding faster than the speed of light?

There is no unique speed associated with the expansion. The scale a is a relative measure, whose fractional rate of increase $(1/a)\ da/dt$ is known as the Hubble parameter, and is in general a function of time. Just to be confusing, the value of H today is referred to as the Hubble constant.

For curved geometries, there is a characteristic *curvature radius* R which increases proportional to a. However, its rate of change dR/dt is not a physical speed: nothing is going anywhere at that speed.

Are there galaxies moving away from us at more than the speed of light?

Yes, but . . .

How do we assign a distance and velocity to a faraway galaxy in an expanding universe? In GR, we measure distances between two events that occur at the same time. But time is relative, and so our choice is not unique. However, there is a choice that reflects the symmetry of spacetime. The matter-energy in the universe defines a rest frame, in which one's (cosmological) surroundings are not moving on average. Clocks at rest in this frame measure *cosmic time* t, and the distance measured at constant cosmic time is known as proper distance r_p. The recession velocity of a galaxy is then dr_p/dt.

In a homogeneous expansion, recession velocity increases proportional to distance. Galaxies at sufficiently large distances recede faster than the speed of light. This doesn't violate special relativity, as it doesn't represent a velocity *through* space. Put another way, you can't use the expansion of space to make two objects pass each other at more than the speed of light.

Light from distant galaxies is observed to be redshifted. Is this because the expansion of space stretches the wavelength, or because is it a Doppler shift due to the recession of the galaxy?

Once again, locally there is no difference. If, since the emission of the photon, the universe doubles in scale, then you'll see a photon with double the emitted wavelength. Thus, we can think of the expansion of space as stretching the wavelength of light.

There is a complication. An alternative way of assigning a velocity to a galaxy is consistent with the special relativistic Doppler relationship between velocity and redshift[1]. This velocity does not break the light-speed limit. I prefer the expanding space picture for the following reason. If the universe is finite and sufficiently small, you could see the back of your own head, redshifted. If you're thinking of redshift as a Doppler shift, are you moving away from the back of your own head?[2]

Does the universe have zero total energy?

Probably not. In Newton's theory, one can define the total energy of the contents of the universe, and the total gravitational energy, which is usually negative. In special cases — in particular, the Newtonian counterpart of a flat universe — those two contributions cancel giving zero total energy. Such a calculation has no place in General Relativity, for several reasons. Firstly, in the absence of a fixed background space and time, one cannot simply add up all the mass-energy in the universe. Secondly, there is no such thing as gravitational energy in GR, because there is no gravitational field. Finally, conserved quantities in physics follow from symmetries. Energy conservation follows from time-translation symmetry. An expanding universe does not exhibit this symmetry, so energy is not globally conserved.[3]

Energy is not conserved!? Shouldn't that send shivers up the spine of any physicist?

This is the dirty secret of cosmology. The most familiar conserved quantities—energy and momentum — are linked to spacetime symmetries. GR messes with the fabric of spacetime itself, and so these symmetries are not to be expected in any old spacetime.

The universe is expanding, so not time-translation symmetric, so doesn't conserve energy. This doesn't mean that chaos reigns. The energy density of the universe changes predictably. We can still do thermodynamics and when we do, especially in relation to the formation of the light elements in the very early universe, we find some of besttested predictions of modern cosmology.

The very universe, we are told, began in thermal equilibrium. How did equilibrium establish itself so quickly?

In conventional cosmology, local thermal equilibrium cannot establish itself in the early universe. This is known as the horizon problem. The light from the early universe (the Cosmic Microwave Background, CMB) has the same temperature in every direction, to 1 part in 100,000. We are seeing parts of the universe that, at the time when they are sending their light to us, have not been in contact with each other. How, then, have they managed to agree about their temperature? We cannot get a universe like ours by starting with a mess and letting thermal effects smooth it out. Homogeneity, isotropy and thermal equilibrium, observed in the CMB, must be assumed as initial conditions.

By "conventional cosmology", I mean a universe dominated by ordinary matter and radiation at its earliest times. We can solve the horizon problem if we posit that the early universe contained a form of energy that caused a very brief but explosive period of accelerated expansion. This process is known as inflation. You'll note the similarity to dark energy, except that we need a new form of energy that not only accelerates the expansion of the universe, but does it for a finite period of time. It needs an off switch. This suggests that it is not the same stuff as dark energy. Inflationary theory makes other successful predictions that give us enough clues to think that maybe it really happened. As with dark energy, find it and collect your Nobel prize.

How does the initially smooth universe we see in the CMB become today's universe of stars and galaxies?

The short story: gravity. A region of the universe that is more dense than average will attract more matter, becoming still more dense. Eventually, galaxies form and themselves combine and merge. The Antennae Galaxies (see Figure 3) are colliding: they will orbit and spiral inwards, their gas shocking and coalescing into stars, until they merge into one large galaxy.

I may have raised more questions than I've answered. If you want to ask a follow up question, then head over to my blog, Letters to Nature: goo.gl/Al2ZqA.

Figure 3: The Antennae Galaxies

References

[1] E.F. Bunn and D.W. Hogg, Am. *J. Phys.*, **77**, 688 (2009)

[2] B.F. Roukema, *MNRAS* **404**, 318 (2010)

[3] Did Lawrence Krauss tell you otherwise? In Chapter 10 of A Universe from Nothing, he admits these points: "[T]he average Newtonian gravitational energy of every object in our flat universe is zero. . . . But this is not the whole story. . . . [E]nergy as we normally think of it elsewhere in physics is not a particularly well-defined concept on large scales in a curved universe. . . . There is a lot of debate over precisely how to [add up energy in an infinite universe]."

AUTHOR BIOGRAPHY

Dr Luke A. Barnes is a postdoctoral researcher at the Sydney Institute for Astronomy. After undergraduate studies at the University of Sydney, Dr. Barnes earned a scholarship to complete a PhD at the University of Cambridge. He worked as a researcher at the Swiss Federal Institute of Technology (ETH), before returning to Sydney in 2011. He has published papers on galaxy formation and cosmology, and recently has taken an interest in the fine-tuning of the universe for intelligent life. He blogs at **letterstonature.wordpress.com.**